Mathematical Work of Federico Rodriguez Hertz

Federico Rodriguez Hertz has made a number of first-rate contributions to the field of dynamical systems. His work contains a number of brilliant innovative insights, as well as technical tours de force. It includes solutions of several well-known problems that have resisted the efforts of the leading experts in the field for a long time. His contributions cover several of the most active and exciting areas of current research: ergodic theory of smooth dynamical systems, theory of partially hyperbolic systems, rigidity of actions of higher rank abelian groups. Due to space limitations we only mention his most significant achievements.

In his Ph.D. thesis that was published in the Annals of Mathematics Rodriguez Hertz made a crucial advance in the stable ergodicity program. A central paradigm of modern dynamics relates global long-term behavior (e.g. ergodicity) with infinitesimal behavior of orbits, it comes in several flavors, principal among them being uniform, non-uniform (in time) and partial (in space) hyperbolicity. That the first class of systems exhibits robust ergodic behavior was established in the classical works of D. Anosov and Ya. Sinai from the 1960's that in turn built on earlier achievements of M. Morse, Hedlund and E. Hopf. Partial hyperbolicity is a more subtle albeit more wide-spread situation. Classical examples come from algebraic models, namely flows and diffeomorphisms of homogeneous spaces induced by one parameter subgroups, and automorphisms of Lie groups. One of the early highlights here is an observation of H. Weyl from the 1920's that the ergodic automorphisms of the n-torus correspond precisely to those integer matrices of determinant one and no eigenvalue a root of unit. Some of these ergodic automorphisms do have eigenvalues of modulus one and so are only partially hyperbolic.

A central thrust in the modern partial hyperbolicity theory whose fundamentals were developed in the 1970's by M.Hirsch, C. Pugh, M. Shub, M. Brin and Ya.Pesin, is to establish prevalence of stable ergodicity, i.e. to show that typically a small volume preserving perturbation of an ergodic partially hyperbolic system remains ergodic. Lots of efforts went into this program and considerable progress has been reached starting from 1994 in the works of Pugh, Shub, M. Grayson, A. Wilkinson and K. Burns. All this work relied on an additional property that can be non-technically described as dominance of hyperbolic directions: other directions can be reached by moving along the hyperbolic ones. Here Weyl's example presented a challenge, since locally hyperbolic directions are locally integrable and hence produce nothing new. No one had an idea how to proceed. In a brilliant, innovative and unexpected thesis Federico Rodriguez Hertz proved stable ergodicity in this case, under a technical assumption that there are two eigenvalues are of unit modulus.

Another major achievement in the stable ergodicity program is the 2008 Inventiones paper joint with M.A. Rodriguez Hertz and R.Ures that completely solves the 1995 Pugh-Shub stable ergodicity conjecture in the case closest to the hyperbolic one: when the neutral (nonhyperbolic) direction is one-dimensional. They show that in this case stably ergodic diffeomorphisms are C^r dense among volume preserving partially hyperbolic systems for any r > 1. Also, joint with M.A. Rodriguez Hertz, A. Tahzibi and R. Ures, Rodriguez Hertz improves the description of the ergodic components in Pesin ergodic decomposition Theorem and use it to proof the Pugh-Shub conjecture when the neutral direction is two-dimensional (r=1) and uniqueness of SRB measures for surface diffeomorphisms.

Another direction in Federico Rodriguez Hertz' work is rigidity theory for smooth group actions on compact manifolds. It is well understood that for classical dynamical systems (diffeomorphisms and flows, i.e. actions of Z or R) only the topological orbit structure can be stable (structural stability) while the differentiable orbit structure is at best described locally by infinitely moduli. In the 1980's R. Zimmer formulated his now classical program (modified in the 1990's after some examples were found by A. Katok and J. Lewis) aimed at establishing that for \large" and \rigid" groups, such as semisimple Lie groups of R- rank greater than one or lattices in such groups, the orbit structure should be quite rigid: in fact built from algebraic blocks. Zimmer's program relies on earlier fundamental work of A. Weil, G.Mostow and G. Margulis. The local version of the Zimmer program has been carried out in the work of many authors, the crowing achievement here being the series of papers by Margulis and D. Fisher. The first advance in the truly global case is in the joint work of Rodriguez Hertz and Katok that is in preparation. It uses the work on non-uniform measure rigidity discussed below. Quite remarkably, in the presence of a certain kind of hyperbolicity the rigidity of the differentiable orbit structure appears for actions of higher rank abelian groups such as Z^k or R^k for k > 1. The local rigidity for the principal classes of algebraic models was established in the 1990's by M. Guysinsky, Katok and R. Spatzier. A breakthrough here is the first truly global result by Rodriguez Hertz that appeared in 2007 in the Journal of Modern Dynamics; it solved a problem formulated as early as 1990.

Another aspect of rigidity for actions of higher rank abelian groups is the relative scarcity of invariant measures for such actions in contrast with the great abundance of such measures for hyperbolic diffeomorphisms and flows. This work goes back to the famous x2; x3 question by Furstenberg. Advances for the case of algebraic actions were made by R. Lyons, D. Rudolph, Katok, Spatzier, E. Lindenstrauss, M. Einsiedler, B. Kalinin, and others. Among applications are advances in quantum unique ergodicity and partial solution of the 1930 Littlewood conjecture in Diophantine approximation. An important and rather unexpected new dimension of this development is non-uniform measure rigidity that has been developed by Rodriguez Hertz in collaboration with Kalinin and Katok. The outcome here is existence of absolutely continuous invariant measures as well as other invariant geometric structures for actions defined by global topological data, such as the homotopy types of its elements. A side effect of this development is an introduction of new powerful tools and insights into smooth ergodic theory.