Mathematical Work of Fernando Codá Marques

Fernando Codá Marques obtained his Ph.D. in 2003 at Cornell University (USA) under the guidance of Professor José F. Escobar. On his return he obtained a permanent position at IMPA. He visited Stanford University (Post-doc) for a year (2005-2006) where he started cooperation with Professor Richard Schoen. He is mainly interested in the application of analytic techiques to the study of geometric problems.

Marques has made brilliant contributions to the Yamabe problem and to geometric flows (scalar curvature). The Yamabe Problem is a generalization of Riemann's Uniformization Theorem to dimensions greater than 2. The goal is to study the set of constant scalar curvature metrics in a given conformal class. The existence problem was solved around 25 years ago, and in 1988 Richard Schoen conjectured that the set of solutions (of volume one) should be always compact, unless the manifold is conformally diffeomorphic to the round sphere. In a joint work with M. Khuri and Richard Schoen [KMS], Marques proved that the Compactness Conjecture is true if the dimension is less than 25. The main contribution was the discovery of a canonical quadratic form for each dimension that regulates the problem. The local pointwise estimates in that paper are based on the methods of Marques's earlier work in dimensions 6 and 7. On the other hand S. Brendle discovered smooth counterexamples to the conjecture in dimensions higher than 51, which were later extended by Brendle and Marques to the remaining dimensions (between 24 and 52). The dimension phenomenon is new and quite surprising, and can be explained by the behavior of the quadratic form found in [KMS]. This series of papers gives a complete answer to Schoen's conjecture.

There are several geometric flows being studied today which are described by parabolic differential equations. The most famous among them is the Ricci flow, introduced by Hamilton in 1982 and used by G. Perelman in his proof of the Poincaré Conjecture. Very recently Marques has become interested in the applications of such flows to the understanding of the topology of natural spaces of metrics. He has been able to use the Ricci flow with surgeries to prove that the space of positive scalar curvature metrics on the 3-sphere is path-connected. The corresponding result for the 2-sphere was proved in 1916 by Hermann Weyl. The result is known to be false for some high-dimensional spheres. Marques also proves that the moduli space of positive scalar curvature metrics of any orientable compact three-manifold is path-connected, provided it is nonempty. The condition of positive scalar curvature is preserved by the Ricci flow but singularities are unavoidable. Marques's approach is based on Perelman's description of the singularities, and consists in introducing the connected sum construction of Gromov and Lawson to undo the surgeries at each singular time. Marques's result has also implications for the deformation of initial data of the Einstein equations in General Relativity.

These two contributions are very impressive. We note that, recently, Fernando Marques is an invited speaker at the International Congress of Mathematicians- ICM 2010, Hyderabad (India).