On a conjecture by De Giorgi in large dimensions

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A celebrated conjecture due to E. De Giorgi [3] states that any bounded solution of the equation $\Delta u + (1-u^2)u = 0$ in $\mathbf{R}^N$ with $\partial_{x_N} u > 0$ must be such that its level sets $\{u = \lambda\}$ are all hyperplanes, *at least* for dimension $N \leq 8$. This conjecture is known to be true in dimensions $N = 2, 3$ [1, 4], and true under a mild extra assumption [5]. Based on a minimal graph Γ which is not a hyperplane, found by Bombieri, De Giorgi and Giusti [2] in $\mathbf{R}^N$, $N \geq 9$, we prove that for any small $\alpha > 0$ there is a bounded solution $u_\alpha(x)$ with $\partial_{x_N} u_\alpha > 0$, which resembles $\tanh\left(\frac{t}{\sqrt{2}}\right)$, where t denotes a choice of signed distance to the blown-up minimal graph $\Gamma_\alpha := \alpha^{-1}\Gamma$. This solution constitutes a "counterexample" to De Giorgi's conjecture for $N \geq 9$. This is joint work with Michal Kowalczyk and Juncheng Wei.

References

[1] L. Ambrosio and X. Cabré, *Entire solutions of semilinear elliptic equations in* $\mathbf{R}^3$ *and a conjecture of De Giorgi,* Journal Amer. Math. Soc. 13 (2000), 725–739.

[2] E. Bombieri, E. De Giorgi, E. Giusti, *Minimal cones and the Bernstein problem,* Invent. Math. 7 1969 243–268.

[3] E. De Giorgi, *Convergence problems for functionals and operators,* Proc. Int. Meeting on Recent Methods in Nonlinear Analysis (Rome, 1978), 131–188, Pitagora, Bologna (1979).

[4] N. Ghoussoub and C. Gui, *On a conjecture of De Giorgi and some related problems,* Math. Ann. 311 (1998), 481–491.

[5] O. Savin, *Regularity of flat level sets in phase transitions.* Ann. of Math. (2) 169(2009), no.1, 41-78.

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