STATISTICAL PROPERTIES OF REAL AND COMPLEX ONE-DIMENSIONAL DYNAMICAL SYSTEMS

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Absolutely continuous invariant measures form a natural source of physical measures for one-dimensional systems. The existence and statistical properties of such measures have been extensively studied in the last few decades. In this talk we will survey the main results in this area of study, for sufficiently regular interval maps, as well as for complex rational maps.

Some of the most prominent results were obtained in the case when the derivatives at critical values grow to infinity sufficiently fast. Somewhat surprisingly, it turns out that it is sufficient that the derivatives a critical values are large to guaranty the existence of an absolutely continuous invariant measure with good statistical properties (having polynomial decay of correlations, and satisfying the central limit theorem.) This result, obtained in collaboration with Weixiao Shen, has been inspired by previous work of, or with, Henk Bruin, Weixiao Shen, and Sebastian van Strien.

We will also review some of the applications of the methods developed in the proof of these results to the understanding of phase transitions, as well as to the understanding of fractal dimensions, and other properties of the Julia set of a real or complex map whose derivatives at critical values are large.