Symmetric and skew-symmetric elements in group algebras

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Let RG denote the group algebra of a group G over a commutative ring with unity R. The map $\sum_{g\in G} a_g g \mapsto \sum_{g\in G} a_g g^{-1}$ is an involution of RG known as its *classical involution*.

More generally, if $^* : G \to G$ is any involution of the group G, the map $\sum_{g\in G} a_g g \mapsto \sum_{g\in G} a_g g^*$ is a *group involution* of RG, which we shall denote again by *.

Consider the sets:

$$RG^+ = \{\alpha \in RG \mid \alpha^* = \alpha\},$$
$$RG^- = \{\alpha \in RG \mid \alpha^* = -\alpha\},$$

of *symmetric* and *skew-symmetric* elements in RG respectively.

We shall give an account of the results obtained in recent years, by several researchers, regarding properties of these sets such as commutativity, anti-commutativity, Lie nilpotence, existence of a group identity, etc. and their relation to other questions, in particular, to conditions for a loop algebra to be alternative. Also, some connections with symmetric and skew-symmetric units will be established.